

Consistent massive truncations of IIB supergravity on Sasaki-Einstein manifolds

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Abstract

Recent work on holographic superconductivity and gravitational duals of systems with non-relativistic conformal symmetry have made use of consistent truncations of $D = 10$ and $D = 11$ supergravity retaining some massive modes in the Kaluza-Klein tower. In this paper we focus on reductions of IIB supergravity to five dimensions on a Sasaki-Einstein manifold, and extend these previous truncations to encompass the entire bosonic sector of gauged $D = 5$, $\mathcal{N} = 2$ supergravity coupled to massive multiplets up to the second Kaluza-Klein level. We conjecture that a necessary condition for the consistency of massive truncations is to only retain the lowest modes in the massive trajectories of the Kaluza-Klein mode decomposition of the original fields. This is an extension of the well-known result that consistent truncations may be obtained by restricting to the singlet sector of the internal symmetry group.

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I. INTRODUCTION

Recent developments in AdS/CFT have expanded the scope of applications from the realm of strongly coupled relativistic gauge theories to various condensed matter systems whose dynamics are expected to be described by a strongly coupled theory. These include systems with behavior governed by a quantum critical point [1, 2], as well as cold atoms and similar systems exhibiting non-relativistic conformal symmetry [3, 4]. Much current attention is also directed towards holographic descriptions of superfluids and superconductors [5–8].

The main feature used in the construction of a dual model of superconductivity is the existence of a charged scalar field in the dual AdS background [6, 8]. Turning on temperature and non-zero chemical potential corresponds to working with a charged black hole in AdS. Then, as the temperature is lowered, the charged scalar develops an instability and condenses, so that the black hole develops scalar hair¹. This condensate breaks the U(1) symmetry, and is a sign of superconductivity (in the case where the U(1) is “weakly gauged” on the boundary).

The basic model dual to a 2+1 dimensional superconductor is simply that of a charged scalar coupled to a Maxwell field and gravity, and may be described by a Lagrangian of the form

$$\mathcal{L}_4 = R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}^2 - |\partial_\mu\psi - iqA_\mu\psi|^2 - m^2|\psi|^2. \quad (1)$$

The properties of the system may then be studied for various values of mass m and charge q . While this is a perfectly acceptable framework, a more complete understanding demands that this somewhat phenomenological Lagrangian be embedded in a more complete theory such as string theory, or at least its supergravity limit. For AdS₄ duals of 2+1 dimensional superconductors, this was examined at the linearized level in [12], and embedded into $D = 11$ supergravity at the full non-linear level in [13–15] for the case $m^2L^2 = -2$ and $q = 2$. Similarly, a IIB supergravity model for an AdS₅ dual to 3+1 dimensional superconductors was constructed in [16] with $m^2L^2 = -3$ and $q = 2$.

The AdS₄ model of [13–15] and the AdS₅ model of [16] are based on Kaluza-Klein trun-

¹ Recent models have generalized this construction to encompass both p-wave [9, 10] and d-wave [11] condensates.

cations on squashed Sasaki-Einstein manifolds. They both have the unusual feature where the $q = 2$ charged scalar arises from the massive level of the Kaluza-Klein truncation. This appears to go against the standard lore of consistent truncations, where it was thought that truncations keeping only a finite number of massive modes would necessarily be inconsistent. A heuristic argument is that states in the Kaluza-Klein tower carry charges under the internal symmetry, and hence would couple at the non-linear level to source higher and higher states, all the way up the Kaluza-Klein tower. This hints that one way to obtain a consistent truncation is simply to truncate to singlets of the internal symmetry group, and indeed such a construction is consistent. An example of this is a standard torus reduction, where only zero modes on the torus are kept. On the other hand, sphere reductions to maximal gauged supergravities in $D = 4, 5$ and 7 do not follow this rule, as they are expected to be consistent, even though some of the lower-dimensional fields (such as the non-abelian graviphotons) are charged under the R -symmetry. In fact, the issue of Kaluza-Klein consistency is not yet fully resolved, and often must be treated on a case by case basis. This has led us to explore the squashed Sasaki-Einstein compactifications to see if additional consistent massive truncations may be found.

In addition to embedding holographic models of superconductivity into string theory, several groups have demonstrated the embedding of dual non-relativistic CFT backgrounds into string theory [17–19]. These geometries were originally constructed from a toy model of a massive vector field coupled to gravity with a negative cosmological constant [3, 4] of the form (given here for a deformation of AdS_5):

$$\mathcal{L}_5 = R + \frac{12}{L^2} - \frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2, \quad (2)$$

with mass related to the scaling exponent z according to $m^2 L^2 = z(z + 2)$. The $z = 2$ and $z = 4$ models ($m^2 L^2 = 8$ and $m^2 L^2 = 24$, respectively) were subsequently realized within IIB supergravity in terms of consistent truncations retaining a massive vector (along with possibly other fields as well) [17–19]. These results have further opened up the possibility of obtaining large classes of consistent truncations retaining massive modes of various spin.

A. Consistent massive truncations of IIB supergravity

For the most part, the massive consistent truncations used in the study of AdS/condensed matter systems have not been supersymmetric². Nevertheless this has motivated us to investigate the possibility of obtaining new supersymmetric massive truncations of IIB supergravity. In particular, we are mainly interested in reducing IIB supergravity on a Sasaki-Einstein manifold to obtain gauged supergravity in $D = 5$ coupled to possibly massive supermultiplets.

Following the construction of $D = 11$ supergravity [20] and the realization that it admits an $\text{AdS}_4 \times S^7$ vacuum solution [21], it was soon postulated that the Kaluza-Klein reduction on the sphere would give rise to gauged $\mathcal{N} = 8$ supergravity at the “massless” Kaluza-Klein level [22–24]. This notion was reinforced by a linearized Kaluza-Klein mode analysis demonstrating that the full spectrum of Kaluza-Klein excitations falls into supermultiplets of the $D = 4$, $\mathcal{N} = 8$ superalgebra $\text{OSp}(4|8)$ [25–27]. However, demonstrating full consistency of the non-linear reduction to gauged $\mathcal{N} = 8$ supergravity has remained elusive. Nevertheless, all indications are that the reduction is consistent [28], and this has in fact been demonstrated for the related case of reducing to $D = 7$ on S^4 [29, 30].

The story is similar for the case of IIB supergravity reduced on S^5 . A linearized Kaluza-Klein mode analysis demonstrates that the spectrum of Kaluza-Klein excitations falls into complete supermultiplets of the $D = 5$, $\mathcal{N} = 8$ superalgebra $\text{SU}(2, 2|4)$, with the lowest one corresponding to the ordinary $\mathcal{N} = 8$ supergravity multiplet [31, 32]. In this case, only partial results are known about the full non-linear reduction to gauged supergravity, but there is strong evidence for its consistency [33–36].

More generally, it was conjectured in [37, 38] and [39], that, for any supergravity reduction, it is always possible to consistently truncate to the supermultiplet containing the massless graviton. This is a non-trivial statement, as the truncation must satisfy rather restrictive consistency conditions related to the gauge symmetries generated by the isometries of the internal manifold [40, 41]. This conjecture has recently been shown to be true for Sasaki-Einstein reductions of IIB supergravity on SE_5 [42] and $D = 11$ supergravity on SE_7 [39], yielding minimal $D = 5$, $\mathcal{N} = 2$ and $D = 4$, $\mathcal{N} = 2$ gauged supergravity, respectively

² The massive truncation given in [13] is supersymmetric, although the connection to a holographic superconductor was done through the non-supersymmetric skew-whiffed case.

(see also [43, 44]).

While states in the same supermultiplet do not necessarily have the same mass in gauged supergravity, the minimal supergravity multiplets, which contain the graviton, gravitino and a graviphoton, are in fact massless. Thus one may suspect that truncations to massless supermultiplets are necessarily consistent. However, it turns out that this is not the case. This was explicitly demonstrated in [41], where, for example, it was shown to be inconsistent to retain the $SU(2) \times SU(2)$ vector multiplets that naturally arise in the compactification of IIB supergravity on $T^{1,1}$.

For many of the above reasons, it has often been a challenge to explore consistent supersymmetric truncations, even at the massless Kaluza-Klein level. However, bosonic truncations retaining massive breathing and squashing modes [45] have been known to be consistent for some time. In this case, consistency is guaranteed by retaining only singlets under the internal symmetry group $SU(4) \times U(1)$ for the squashed S^7 or $SU(3) \times U(1)$ for the squashed S^5 . The supersymmetry of background solutions involving the breathing and squashing modes was explored in [46], where it was further conjectured that a supersymmetric consistent truncation could be found that retains the full breathing/squashing supermultiplet.

Although this massive consistent truncation conjecture was made for squashed sphere compactifications, it naturally generalizes to compactification on more general internal spaces, such as Sasaki-Einstein spaces. For $D = 11$ supergravity compactified on a squashed S^7 , written as $U(1)$ bundled over CP^3 , truncation of the $\mathcal{N} = 8$ Kaluza-Klein spectrum to $SU(4)$ singlets under the decomposition $SO(8) \supset SU(4) \times U(1)$ yields the $\mathcal{N} = 2$ supergravity multiplet³

$$n = 0 : \quad \mathcal{D}(2, 1)_0 = D(3, 2)_0 + D(\tfrac{5}{2}, \tfrac{3}{2})_{-1} + D(\tfrac{5}{2}, \tfrac{3}{2})_1 + D(2, 1)_0, \quad (3)$$

at the massless ($n = 0$) Kaluza-Klein level. No $SU(4)$ singlets survive at the first ($n = 1$) massive Kaluza-Klein level, and the breathing and squashing modes finally make their appearance at the second ($n = 2$) Kaluza-Klein level in a massive vector multiplet [46]

$$\begin{aligned} n = 2 : \quad \mathcal{D}(4, 0)_0 = & D(5, 1)_0 + D(\tfrac{9}{2}, \tfrac{1}{2})_{-1} + D(\tfrac{9}{2}, \tfrac{1}{2})_1 + D(\tfrac{11}{2}, \tfrac{1}{2})_{-1} + D(\tfrac{11}{2}, \tfrac{1}{2})_1 \\ & + D(4, 0)_0 + D(5, 0)_0 + D(5, 0)_{-2} + D(5, 0)_2 + D(6, 0)_0. \end{aligned} \quad (4)$$

³ The $OSp(4|2)$ super-representations $\mathcal{D}(E_0, s)_q$ and $SO(2, 3)$ representations $D(E_0, s)_q$ are labeled by energy E_0 , spin s and $U(1)$ charge q under $OSp(4|2) \supset SO(2, 3) \times U(1) \supset SO(2) \times SO(3) \times U(1)$.

Replacing S^7 by SE_7 amounts to replacing CP^3 by an appropriate Kahler-Einstein base B . In this case, the internal isometry is generically reduced from $SU(4) \times U(1)$. Nevertheless, the notion of truncating to $SU(4)$ singlets may simply be replaced by the prescription of truncating to zero modes on the base B . This procedure was in fact done in [13], which constructed the non-linear Kaluza-Klein reduction for all the bosonic fields contained in the above supermultiplets (3) and (4) and furthermore verified the $\mathcal{N} = 2$ supersymmetry.

For the case of IIB supergravity compactified on SE_5 , it is straightforward to generalize the squashed S^5 conjecture of [46]. In this case, however, the Kaluza-Klein spectrum is more involved, and is given in Table I. A curious feature shows up here in that an additional LH+RH chiral matter multiplet shows up at the ‘massless’ Kaluza-Klein level. The $E_0 = 4$ scalar in this multiplet corresponds to the IIB axi-dilaton, while the additional $E_0 = 3$ charged scalar is precisely the charged scalar constructed in the holographic model of [16]. At the higher Kaluza-Klein levels, the breathing and squashing mode scalars correspond to the $E_0 = 8$ and $E_0 = 6$ scalars in the massive vector multiplet. In addition, consistent truncations involving the $E_0 = 5$ ($m^2 L^2 = 8$) doublet of vectors in the semi-long LH+RH massive gravitino multiplet and the $E_0 = 7$ ($m^2 L^2 = 24$) vector in the massive vector multiplet were constructed in [17–19] in the context of investigating non-relativistic conformal backgrounds in string theory.

What we have seen so far is that massive consistent truncations of IIB supergravity have been obtained keeping various subsets of the bosonic fields identified in Table I. The goal of this paper is to construct a complete non-linear Kaluza-Klein reduction of IIB supergravity on SE_5 retaining all the bosonic fields in the multiplets up to the $n = 2$ level. This complements the massive Kaluza-Klein truncation of $D = 11$ supergravity [13], and provides another example of a consistent truncation retaining the breathing mode supermultiplet. We proceed in Section II with the Sasaki-Einstein reduction of IIB supergravity. Then in Section III we connect the full non-linear reduction with the linearized Kaluza-Klein analysis of [31, 32] and show how the bosonic fields in Table I are related to the original IIB fields. In Section IV we relate the complete non-linear reduction to previous results by performing additional truncations to a subset of active fields. Finally, we conclude in Section V with some further speculation on massive consistent truncations of supergravity.

While this work was being completed we became aware of [47–49] which independently worked out the massive consistent truncation of IIB supergravity on SE_5 .

n	Multiplet	$SU(2, 2 1)$	$SO(2, 4) \times U(1)$
0	supergraviton	$\mathcal{D}(3, \frac{1}{2}, \frac{1}{2})_0$	$D(4, 1, 1)_0 + D(3\frac{1}{2}, 1, \frac{1}{2})_{-1} + D(3\frac{1}{2}, \frac{1}{2}, 1)_1 + D(3, \frac{1}{2}, \frac{1}{2})_0$
0	LH chiral	$\mathcal{D}(3, 0, 0)_2$	$D(3\frac{1}{2}, \frac{1}{2}, 0)_1 + D(3, 0, 0)_2 + D(4, 0, 0)_0$
0	RH chiral	$\mathcal{D}(3, 0, 0)_{-2}$	$D(3\frac{1}{2}, 0, \frac{1}{2})_{-1} + D(3, 0, 0)_{-2} + D(4, 0, 0)_0$
1	LH massive gravitino	$\mathcal{D}(4\frac{1}{2}, 0, \frac{1}{2})_1$	$D(5\frac{1}{2}, \frac{1}{2}, 1)_1 + D(5, \frac{1}{2}, \frac{1}{2})_0 + D(5, 0, 1)_2$ $+ D(6, 0, 1)_0 + D(4\frac{1}{2}, 0, \frac{1}{2})_1 + D(5\frac{1}{2}, 0, \frac{1}{2})_{-1}$
1	RH massive gravitino	$\mathcal{D}(4\frac{1}{2}, \frac{1}{2}, 0)_{-1}$	$D(5\frac{1}{2}, 1, \frac{1}{2})_{-1} + D(5, \frac{1}{2}, \frac{1}{2})_0 + D(5, 1, 0)_{-2}$ $+ D(6, 1, 0)_0 + D(4\frac{1}{2}, \frac{1}{2}, 0)_{-1} + D(5\frac{1}{2}, \frac{1}{2}, 0)_1$
2	massive vector	$\mathcal{D}(6, 0, 0)_0$	$D(7, \frac{1}{2}, \frac{1}{2})_0 + D(6\frac{1}{2}, \frac{1}{2}, 0)_{-1} + D(6\frac{1}{2}, 0, \frac{1}{2})_1$ $+ D(7\frac{1}{2}, 0, \frac{1}{2})_{-1} + D(7\frac{1}{2}, \frac{1}{2}, 0)_1 + D(6, 0, 0)_0$ $+ D(7, 0, 0)_{-2} + D(7, 0, 0)_2 + D(8, 0, 0)_0$

TABLE I: The truncated Kaluza-Klein spectrum of IIB supergravity on squashed S^5 [46], or equivalently on SE_5 . Here n denotes the Kaluza-Klein level. The consistent truncation is expected to terminate at level $n = 2$ with the breathing mode supermultiplet.

II. SASAKI-EINSTEIN REDUCTION OF IIB SUPERGRAVITY

The bosonic field content of IIB supergravity consists of the NSNS fields (g_{MN}, B_{MN}, ϕ) and the RR potentials (C_0, C_2, C_4) . Because of the self-dual field strength $F_5^+ = dC_4$, it is not possible to write down a covariant action. However, we may take a bosonic Lagrangian of the form

$$\mathcal{L}_{\text{IIB}} = R * 1 - \frac{1}{2\tau_2^2} d\tau \wedge *d\bar{\tau} - \frac{1}{2} \mathcal{M}_{ij} F_3^i \wedge *F_3^j - \frac{1}{4} \tilde{F}_5 \wedge * \tilde{F}_5 - \frac{1}{4} \epsilon_{ij} C_4 \wedge F_3^i \wedge F_3^j, \quad (5)$$

where self-duality $\tilde{F}_5 = * \tilde{F}_5$ is to be imposed by hand after deriving the equations of motion.

We have given the Lagrangian in an $SL(2, \mathcal{R})$ invariant form where

$$\tau = C_0 + ie^{-\phi}, \quad \mathcal{M} = \frac{1}{\tau_2} \begin{pmatrix} |\tau|^2 & -\tau_1 \\ -\tau_1 & 1 \end{pmatrix}, \quad (6)$$

and where

$$F_3^i = dB_2^i, \quad B_2^i = \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}, \quad \tilde{F}_5 = dC_4 + \frac{1}{2} \epsilon_{ij} B_2^i \wedge dB_2^j. \quad (7)$$

The equations of motion following from (5) and the self-duality of \tilde{F}_5 are

$$\begin{aligned} d\tilde{F}_5 &= \frac{1}{2}\epsilon_{ij}F_3^i \wedge F_3^j, & \tilde{F}_5 &= *\tilde{F}_5, \\ d(\mathcal{M}_{ij} * F_3^j) &= -\epsilon_{ij}\tilde{F}_5 \wedge F_3^j, \\ \frac{d * d\tau}{\tau_2} + i \frac{d\tau \wedge *d\tau}{\tau_2^2} &= -\frac{i}{2\tau_2} G_3 \wedge *G_3, \end{aligned} \quad (8)$$

and the Einstein equation (in Ricci form)

$$\begin{aligned} R_{MN} &= \frac{1}{2\tau_2^2} \partial_{(M} \tau \partial_{N)} \bar{\tau} + \frac{1}{4} \mathcal{M}_{ij} \left(F_{MPQ}^i F_N^{jPQ} - \frac{1}{12} g_{MN} F_{PQR}^i F^{jPQR} \right) \\ &\quad + \frac{1}{4 \cdot 4!} \tilde{F}_{MPQRS} \tilde{F}_N^{PQRS}. \end{aligned} \quad (9)$$

In the above we have introduced the complex three-form $G_3 = F_3^2 - \tau F_3^1$. If desired, this allows us to rewrite the three-form equation of motion as

$$d * G = -i \frac{d\tau}{2\tau_2} \wedge *(G_3 + \bar{G}_3) + i \tilde{F}_5 \wedge G_3. \quad (10)$$

A. The reduction ansatz

Before writing out the reduction ansatz, we note a few key features of Sasaki-Einstein manifolds. A Sasaki-Einstein manifold has a preferred U(1) isometry related to the Reeb vector. This allows us to write the metric as a U(1) fibration over a Kahler-Einstein base B

$$ds^2(SE_5) = ds^2(B) + (d\psi + \mathcal{A})^2, \quad (11)$$

where $d\mathcal{A} = 2J$ with J the Kahler form on B . Moreover, B admits an SU(2) structure defined by the (1,1) and (2,0) forms J and Ω satisfying

$$J \wedge \Omega = 0, \quad \Omega \wedge \bar{\Omega} = 2J \wedge J = 4 *_4 1, \quad *_4 J = J, \quad *_4 \Omega = \Omega, \quad (12)$$

as well as

$$dJ = 0, \quad d\Omega = 3i(d\psi + \mathcal{A}) \wedge \Omega. \quad (13)$$

Note that we are taking the ‘unit radius’ Einstein condition $R_{ij} = 4g_{ij}$ on the Sasaki-Einstein manifold, which corresponds to $R_{ab} = 6g_{ab}$ on the Kahler-Einstein base.

For the reduction, we write down the most general decomposition of the bosonic IIB fields consistent with the isometries of B . For the metric, we take

$$ds_{10}^2 = e^{2A} ds_5^2 + e^{2B} ds^2(B) + e^{2C} (\eta + A_1)^2, \quad (14)$$

where $\eta = d\psi + \mathcal{A}$. Since A_1 gauges the $U(1)$ isometry, it will be related to the $D = 5$ graviphoton. Note, however, that the graviphoton receives additional contributions from the five-form.

The three-form and five-form field strengths can be expanded in a basis of invariant tensors on B . For the three-forms, we work with the potentials

$$B_2^i = b_2^i + b_1^i \wedge (\eta + A_1) + b_0^i \Omega + \bar{b}_0^i \bar{\Omega}. \quad (15)$$

The scalars b_0^i are complex, while the remaining fields are real. Note that we do not include a term of the form $\tilde{b}_0^i J$ in the ansatz, as this field will act simply as a Stückelburg field in the five-dimensional theory. In particular, it does not give rise to any new dynamics in the equations of motion as it can be repackaged as a total derivative plus terms which would simply shift b_2^i and b_1^i ,

$$2\tilde{b}_0^i J = d(\tilde{b}_0^i \wedge (\eta + A_1)) - d\tilde{b}_0^i \wedge (\eta + A_1) - \tilde{b}_0^i F_2. \quad (16)$$

Taking $F_3^i = dB_2^i$ gives

$$\begin{aligned} F_3^i = & (db_2^i - b_1^i \wedge F) + db_1^i \wedge (\eta + A_1) - 2b_1^i \wedge J + Db_0^i \wedge \Omega + D\bar{b}_0^i \wedge \bar{\Omega} \\ & + 3ib_0^i \Omega \wedge (\eta + A_1) - 3i\bar{b}_0^i \bar{\Omega} \wedge (\eta + A_1), \end{aligned} \quad (17)$$

where D is the $U(1)$ gauge covariant derivative

$$Db_0^i = db_0^i - 3iA_1 b_0^i. \quad (18)$$

For convenience, we write this as

$$F_3^i = g_3^i + g_2^i \wedge (\eta + A_1) + g_1^i \wedge J + f_1^i \wedge \Omega + \bar{f}_1^i \wedge \bar{\Omega} + f_0^i \wedge \Omega \wedge (\eta + A_1) + \bar{f}_0^i \wedge \bar{\Omega} \wedge (\eta + A_1), \quad (19)$$

where our notation is such that the g^i 's are real and the f^i 's are complex.

For the self-dual five-form, we take

$$\tilde{F}_5 = (1 + *)[(4 + \phi_0) *_4 1 \wedge (\eta + A_1) + \mathbb{A}_1 \wedge *_4 1 + p_2 \wedge J \wedge (\eta + A_1) + q_2 \wedge \Omega \wedge (\eta + A_1) + \bar{q}_2 \wedge \bar{\Omega} \wedge (\eta + A_1)], \quad (20)$$

where $*_4 1$ denotes the volume form on the Kahler-Einstein base B . Note that we have pulled out a constant background component

$$\tilde{F}_5 = 4(1 + *)\text{vol}(SE_5), \quad (21)$$

which sets up the Freund-Rubin compactification⁴. The two-forms q_2 are complex, while the other fields are real. For later convenience, we take the explicit 10-dimensional dual in the metric (14) to obtain

$$\begin{aligned}\tilde{F}_5 = & (4 + \phi_0) *_4 1 \wedge (\eta + A_1) + \mathbb{A}_1 \wedge *_4 1 + p_2 \wedge J \wedge (\eta + A_1) + q_2 \wedge \Omega \wedge (\eta + A_1) \\ & + \bar{q}_2 \wedge \bar{\Omega} \wedge (\eta + A_1) + e^{5A-4B-C} (4 + \phi_0) * 1 - e^{3A-4B+C} * \mathbb{A}_1 \wedge (\eta + A_1) \\ & + e^{A-C} * p_2 \wedge J + e^{A-C} * q_2 \wedge \Omega + e^{A-C} * \bar{q}_2 \wedge \bar{\Omega},\end{aligned}\tag{22}$$

where $*$ now denotes the Hodge dual in the $D = 5$ spacetime.

B. Reduction of the equations of motion

In order to obtain the reduction, it is now simply a matter of inserting the above decompositions into the IIB equations of motion. The \tilde{F}_5 equation yields

$$\begin{aligned}d(e^{A-C} * p_2) &= 2e^{3A-4B+C} * \mathbb{A}_1 - p_2 \wedge F_2 + \epsilon_{ij} g_1^i \wedge g_3^j, \\ Dq_2 &= 3ie^{A-C} * q_2 + \epsilon_{ij} (f_1^i \wedge g_2^j - f_0^i g_3^j),\end{aligned}\tag{23}$$

along with the constraints

$$\begin{aligned}\phi_0 &= -\frac{2i}{3} \epsilon_{ij} (f_0^i \bar{f}_0^j - \bar{f}_0^i f_0^j), \\ p_2 &= \frac{1}{4} \epsilon_{ij} g_1^i \wedge g_1^j - d[A_1 + \frac{1}{4} \mathbb{A}_1 + \frac{i}{6} \epsilon_{ij} (f_0^i \bar{f}_1^j - \bar{f}_0^i f_1^j)].\end{aligned}\tag{24}$$

The implication of this is that \tilde{F}_5 gives rise to two physical $D = 5$ fields, namely a massive vector \mathbb{A}_1 and a complex antisymmetric tensor q_2 satisfying an odd-dimensional self-duality equation and with $m^2 = 9$. The mass of \mathbb{A}_1 is not directly apparent from (23) as it mixes with A_1 from the metric to yield the massless graviphoton as well as a $m^2 = 24$ massive vector.

⁴ For simplicity, we have assumed a unit radius ($L = 1$) compactification.

The F_3^i equation yields

$$\begin{aligned}
D(e^{3A+C} \mathcal{M}_{ij} * f_1^j) &= -3ie^{5A-C} \mathcal{M}_{ij} f_0^j * 1 + \epsilon_{ij}[(4 + \phi_0)e^{5A-4B-C} f_0^j * 1 - q_2 \wedge g_3^j \\
&\quad + e^{A-C} * q_2 \wedge g_2^j + e^{3A-4B+C} * \mathbb{A}_1 \wedge f_1^j], \\
d(e^{A+4B-C} \mathcal{M}_{ij} * g_2^j) &= \mathcal{M}_{ij}[e^{-A+4B+C} * g_3^j \wedge F + 4e^{3A+C} * g_1^j] \\
&\quad + \epsilon_{ij}[-2e^{A-C} * p_2 \wedge g_1^j - \mathbb{A}_1 \wedge g_3^j - 4e^{A-C}(*q_2 \wedge \bar{f}_1^j + *\bar{q}_2 \wedge f_1^j)], \\
d(e^{-A+4B+C} \mathcal{M}_{ij} * g_3^j) &= \epsilon_{ij}[-(4 + \phi_0)g_3^j + \mathbb{A}_1 \wedge g_2^j - 2p_2 \wedge g_1^j - 4(q_2 \wedge \bar{f}_1^j + \bar{q}_2 \wedge f_1^j) \\
&\quad + 4e^{A-C}(\bar{f}_0^j * q_2 + f_0^j * \bar{q}_2)]. \tag{25}
\end{aligned}$$

These correspond to a pair of charged scalars f_0^i , a pair of $m^2 = 8$ massive vectors g_1^i and a pair of massive antisymmetric tensors b_2^i .

The ten-dimensional Einstein equation (9) reduces to a five-dimensional Einstein equation, as well as the equations of motion for the breathing and squashing modes B and C and the graviphoton A_1 . In particular, in the natural vielbein basis, the frame components of the ten-dimensional Ricci tensor corresponding to the reduction (14) are given by

$$\begin{aligned}
{}^{10}R_{\alpha\beta} &= e^{-2A}[R_{\alpha\beta} - \nabla_\alpha \nabla_\beta (3A + 4B + C) - \eta_{\alpha\beta} \partial_\gamma A \partial^\gamma (3A + 4B + C) - \eta_{\alpha\beta} \square A \\
&\quad + 3\partial_\alpha A \partial_\beta A - 4\partial_\alpha B \partial_\beta B - \partial_\alpha C \partial_\beta C + 4(\partial_\alpha A \partial_\beta B + \partial_\alpha B \partial_\beta A) \\
&\quad + (\partial_\alpha A \partial_\beta C + \partial_\alpha C \partial_\beta A)] - \frac{1}{2}e^{2C-4A} F_{\alpha\gamma} F_{\beta}{}^\gamma, \\
{}^{10}R_{ab} &= \delta_{ab}[6e^{-2B} - 2e^{2C-4B} - e^{-2A}(\square B + \partial_\gamma B \partial^\gamma (3A + 4B + C))], \\
{}^{10}R_{99} &= 4e^{2C-4B} + \frac{1}{4}e^{2C-4A} F_{\gamma\delta} F^{\gamma\delta} - e^{-2A}(\square C + \partial_\gamma C \partial^\gamma (3A + 4B + C)), \\
{}^{10}R_{\alpha 9} &= \frac{1}{2}e^{C-3A}[\nabla^\gamma F_{\alpha\gamma} + F_{\alpha\gamma} \partial^\gamma (A + 4B + 3C)]. \tag{26}
\end{aligned}$$

The α and β indices correspond to the $D = 5$ spacetime, while a and b correspond to the Kahler-Einstein base B and 9 corresponds to the U(1) fiber direction. The covariant derivatives and frame indices on the right hand side of these quantities are with respect to the $D = 5$ metric. In order to reduce to the $D = 5$ Einstein frame metric, we now choose $3A + 4B + C = 0$, or

$$A = -\frac{4}{3}B - \frac{1}{3}C. \tag{27}$$

For convenience, we will retain A in the expressions below. However, it is not independent, and should always be thought of as a shorthand for (27).

Equating the ten-dimensional Ricci tensor (26) to the stress tensor formed out of F_3^i and

\tilde{F}_5 of (19) and (22), we obtain the $D = 5$ Einstein equation

$$\begin{aligned}
R_{\alpha\beta} = & \frac{1}{3}\eta_{\alpha\beta}(-24e^{2A-2B} + 4e^{5A+3C} + \frac{1}{2}e^{8A}(4 + \phi_0)^2) + \frac{28}{3}\partial_\alpha B\partial_\beta B + \frac{8}{3}\partial_{(\alpha}B\partial_{\beta)}C \\
& + \frac{4}{3}\partial_\alpha C\partial_\beta C + \frac{1}{2\tau_2^2}\partial_{(\alpha}\tau\partial_{\beta)}\bar{\tau} + \frac{1}{2}e^{2C-2A}(F_{\alpha\gamma}F_\beta{}^\gamma - \frac{1}{6}\eta_{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}) + \frac{1}{2}e^{-8B}\mathbb{A}_\alpha\mathbb{A}_\beta \\
& + e^{A-C}[(p_{\alpha\gamma}p_\beta{}^\gamma - \frac{1}{6}\eta_{\alpha\beta}p_{\gamma\delta}p^{\gamma\delta}) + 4(q_{(\alpha}{}^\gamma\bar{q}_{\beta)\gamma} - \frac{1}{6}\eta_{\alpha\beta}q_{\gamma\delta}\bar{q}^{\gamma\delta})] \\
& + \mathcal{M}_{ij}[\frac{2}{3}e^{5A-C}\eta_{\alpha\beta}(f_0^i\bar{f}_0^j + \bar{f}_0^i f_0^j) + \frac{1}{2}e^{-2A-2C}(g_{\alpha\gamma}^i g_\beta{}^j - \frac{1}{6}\eta_{\alpha\beta}g_{\gamma\delta}^i g^{j\gamma\delta}) \\
& + \frac{1}{4}e^{-4A}(g_{\alpha\gamma\delta}^i g_\beta{}^{j\gamma\delta} - \frac{2}{9}\eta_{\alpha\beta}g_{\gamma\delta\epsilon}^i g^{j\gamma\delta\epsilon}) + e^{-4B}(g_\alpha^i g_\beta^j + 2(f_\alpha^i \bar{f}_\beta^j + \bar{f}_\alpha^i f_\beta^j))], \tag{28}
\end{aligned}$$

as well as the B , C and A_1 equations of motion

$$\begin{aligned}
d * dB = & [6e^{2A-2B} - 2e^{5A+3C} - \frac{1}{4}e^{8A}(4 + \phi_0)^2] * 1 - \frac{1}{4}e^{-8B}\mathbb{A}_1 \wedge * \mathbb{A}_1 \\
& + \mathcal{M}_{ij}[\frac{1}{8}e^{-2A-2C}g_2^i \wedge * g_2^j + \frac{1}{8}e^{-4A}g_3^i \wedge * g_3^j - \frac{1}{2}e^{5A-C}(f_0^i \bar{f}_0^j + \bar{f}_0^i f_0^j) * 1 \\
& - \frac{1}{4}e^{-4B}(g_1^i \wedge * g_1^j + 2(f_1^i \wedge * \bar{f}_1^j + \bar{f}_1^i \wedge * f_1^j))], \\
d * dC = & [4e^{5A+3C} - \frac{1}{4}e^{8A}(4 + \phi_0)^2] * 1 + \frac{1}{2}e^{2C-2A}F_2 \wedge * F_2 + \frac{1}{4}e^{-8B}\mathbb{A}_1 \wedge * \mathbb{A}_1 \\
& - \frac{1}{2}e^{A-C}(p_2 \wedge * p_2 + 4q_2 \wedge * \bar{q}_2) + \mathcal{M}_{ij}[-\frac{3}{8}e^{-2A-2C}g_2^i \wedge * g_2^j \\
& + \frac{1}{8}e^{-4A}g_3^i \wedge * g_3^j - \frac{3}{2}e^{5A-C}(f_0^i \bar{f}_0^j + \bar{f}_0^i f_0^j) * 1 \\
& + \frac{1}{4}e^{-4B}(g_1^i \wedge * g_1^j + 2(f_1^i \wedge * \bar{f}_1^j + \bar{f}_1^i \wedge * f_1^j))], \\
d(e^{2C-2A} * F_2) = & (4 + \phi_0)e^{-8B} * \mathbb{A}_1 - p_2 \wedge p_2 - 4q_2 \wedge \bar{q}_2 \\
& + \mathcal{M}_{ij}[4e^{-4B} * (f_0^i \bar{f}_1^j + \bar{f}_0^i f_1^j) + e^{-4A} * g_3^i \wedge g_2^j]. \tag{29}
\end{aligned}$$

Note that, in order to obtain the $D = 5$ Einstein equation, we had to shift the reduction of ${}^{10}R_{\alpha\beta}$ an appropriate combination of ${}^{10}R_{ab}$ and ${}^{10}R_{99}$ in order to remove the $\eta_{\alpha\beta}\Box A$ component in the first line of (26).

The IIB equations of motion thus reduce to (23), (25), (28) and (29) as well as the axi-dilaton equation, which we have not written down explicitly, but which will be shown to be consistent below.

C. The effective five-dimensional Lagrangian

We now wish to construct an effective $D = 5$ Lagrangian which reproduces the above equations of motion. This may be done by noting that the $D = 5$ Einstein equation (28)

arises naturally from a Lagrangian of the form

$$\begin{aligned}
\mathcal{L} = & R * 1 + (24e^{2A-2B} - 4e^{5A+3C} - \frac{1}{2}e^{8A}(4 + \phi_0)^2) * 1 - \frac{28}{3}dB \wedge *dB - \frac{8}{3}dB \wedge *dC \\
& - \frac{4}{3}dC \wedge *dC - \frac{1}{2\tau_2^2}d\tau \wedge *d\bar{\tau} - \frac{1}{2}e^{2C-2A}F_2 \wedge *F_2 - \frac{1}{2}e^{-8B}\mathbb{A}_1 \wedge *\mathbb{A}_1 \\
& - e^{A-C}(p_2 \wedge *p_2 + 4q_2 \wedge *\bar{q}_2) + \mathcal{M}_{ij}[-2e^{5A-C}(f_0^i \bar{f}_0^j + \bar{f}_0^i f_0^j) * 1 \\
& - \frac{1}{2}e^{-2A-2C}g_2^i \wedge *g_2^j - \frac{1}{2}e^{-4A}g_3^i \wedge *g_3^j - e^{-4B}(g_1^i \wedge *g_1^j + 2(f_1^i \wedge *\bar{f}_1^j + \bar{f}_1^i \wedge *f_1^j))] \\
& + \mathcal{L}_{CS}.
\end{aligned} \tag{30}$$

We have included a Chern-Simons piece \mathcal{L}_{CS} which cannot be determined from the Einstein equation.

It is now possible to verify that (30) reproduces all the terms in the equations of motion (23), (25) and (29) involving the metric (*ie* the Hodge $*$). The remaining terms may be obtained from the addition of the topological piece

$$\begin{aligned}
\mathcal{L}_{CS} = & \frac{2i}{3}(q_2 \wedge d\bar{q}_2 - \bar{q}_2 \wedge dq_2) - 4A_1 \wedge q_2 \wedge \bar{q}_2 + 2\epsilon_{ij}b_2^i \wedge db_2^j \\
& + \frac{4i}{3}[(\bar{q}_2 - \frac{i}{6}\epsilon_{ij}\bar{f}_0^i g_2^j) \wedge \epsilon_{kl}(f_1^k \wedge g_2^l - f_0^k g_3^l) - (q_2 + \frac{i}{6}\epsilon_{ij}f_0^i g_2^j) \wedge \epsilon_{kl}(\bar{f}_1^k \wedge g_2^l - \bar{f}_0^k g_3^l)] \\
& - A_1 \wedge (p_2 - \frac{1}{4}\epsilon_{ij}g_1^i \wedge g_1^j) \wedge (p_2 - \frac{1}{4}\epsilon_{kl}g_1^k \wedge g_1^l) \\
& - 2[\frac{1}{4}\mathbb{A}_1 + \frac{i}{6}\epsilon_{ij}(f_0^i \bar{f}_1^j - \bar{f}_0^i f_1^j)] \wedge \epsilon_{kl}(g_1^k \wedge g_3^l - \frac{1}{4}g_1^k \wedge g_1^l \wedge F_2).
\end{aligned} \tag{31}$$

Here we recall the definitions

$$f_0^i = 3ib_0^i, \quad f_1^i = Db_0^i, \quad g_1^i = -2b_1^i, \quad g_2^i = db_1^i, \quad g_3^i = db_2^i - b_1^i \wedge F_2, \tag{32}$$

implicit in (17) and (19). Furthermore, ϕ_0 and p_2 are given by (24). Note that, while \mathbb{A}_1 is massive, and does not have a gauge invariance associated with it, it is natural to make the shift

$$\mathbb{A}_1 \rightarrow \mathbb{A}'_1 - \frac{2i}{3}\epsilon_{ij}(f_0^i \bar{f}_1^j - \bar{f}_0^i f_1^j), \tag{33}$$

so that

$$p_2 = \frac{1}{4}\epsilon_{ij}g_1^i \wedge g_1^j - F_2 - \frac{1}{4}\mathbb{F}'_2, \tag{34}$$

where $\mathbb{F}'_2 = d\mathbb{A}'_1$.

We now turn to the axi-dilaton equation obtained from (30). Since τ only shows up in the kinetic term and in \mathcal{M}_{ij} , we see that the τ equation of motion obtained from the $D = 5$ Lagrangian reproduces that obtained from the original IIB Lagrangian. This is because the quantity in the square brackets multiplying \mathcal{M}_{ij} in (30) is the straightforward reduction of $-\frac{1}{2}F_3^i \wedge *F_3^j$ in the original IIB Lagrangian (5).

III. MATCHING THE LINEARIZED KALUZA-KLEIN ANALYSIS

The complete $D = 5$ Lagrangian, as given by (30) and (31), is somewhat opaque. Thus in this section, we demonstrate that it in fact contains the fields corresponding to the Kaluza-Klein mass spectrum noted in Table I. To do this, it is sufficient to look at the linearized level. We first note that the effective $D = 5$ fields are the complex scalars (τ, b_0^i) , real scalars (B, C) , one-form potentials $(A_1, b_1^i, \mathbb{A}_1)$, pair of real two-forms (b_2^i) , the complex two-form (q_2) , and of course the metric $(g_{\mu\nu})$. The $D = 5$ equations of motion (23), (25) and (29) may be linearized on the matter fields to obtain the set

$$\begin{aligned}
d * db_0^i &= (9\delta_j^i + 12i\mathcal{N}^i_j)b_0^j * 1, \\
d * db_1^i &= -8 * b_1^i, \\
d * db_2^i &= -4\mathcal{N}^i_j db_2^j, \\
dq_2 &= 3i * q_2, \\
d * F_2 &= 4 * \mathbb{A}_1, \quad d * F_2 + \frac{1}{4}d * \mathbb{F}_2 = -2 * \mathbb{A}_1, \\
d * dB &= 4(7B + C) * 1, \quad d * dC = 16(B + C) * 1.
\end{aligned} \tag{35}$$

Here we have introduced

$$\mathcal{N} = \mathcal{M}^{-1}\epsilon = \frac{1}{\tau_2} \begin{pmatrix} -\tau_1 & 1 \\ -|\tau|^2 & \tau_1 \end{pmatrix}, \tag{36}$$

with eigenvalues $+i$ and $-i$, corresponding to eigenvectors $\begin{pmatrix} 1 & \tau \end{pmatrix}^T$ and $\begin{pmatrix} 1 & \bar{\tau} \end{pmatrix}^T$, respectively.

The first equation in (35) then decomposes into a pair of equations for the complex scalars $b_0^{m^2=-3}$ and $b_0^{m^2=21}$ with masses $m^2 = -3$ and $m^2 = 21$ according to

$$b_0^i = \begin{pmatrix} 1 \\ \tau \end{pmatrix} b_0^{m^2=-3} + \begin{pmatrix} 1 \\ \bar{\tau} \end{pmatrix} b_0^{m^2=21}. \tag{37}$$

The second equation is that of an $\text{SL}(2, \mathbb{R})$ doublet of real vectors b_1^i with mass $m^2 = 8$. The third equation can be converted to an odd-dimensional self-duality equation [50] $db_2^i = 4\mathcal{N}^i_j * b_2^j$, for a doublet of antisymmetric tensors b_2^i with mass $m^2 = 16$. The fourth equation is already in odd-dimensional self-duality form, and shows that the complex antisymmetric tensor q_2 has mass $m^2 = 9$.

The vector equations can be diagonalized

$$d * (F_2 + \frac{1}{6}\mathbb{F}_2) = 0, \quad d * \mathbb{F}_2 = -24 * \mathbb{A}_1, \tag{38}$$

to identify the massless graviphoton $A_1 + \frac{1}{6}\mathbb{A}_1$ and the massive $m^2 = 24$ vector \mathbb{A}_1 . Finally the B and C equations may be diagonalized to identify the $m^2 = 32$ breathing and $m^2 = 12$ squashing modes

$$d * d\rho = 32\rho * 1, \quad d * d\sigma = 12\sigma * 1, \quad (39)$$

where

$$B = \rho + \frac{1}{2}\sigma, \quad C = \rho - 2\sigma. \quad (40)$$

It is now possible to see how the above linearized modes are organized into $\mathcal{N} = 2$ supermultiplets. As shown in Table I, at the zeroth Kaluza-Klein level, we have the graviton supermultiplet

$$\mathcal{D}(3, \frac{1}{2}, \frac{1}{2})_0 = D(4, 1, 1)_0 + D(3\frac{1}{2}, 1, \frac{1}{2})_{-1} + D(3\frac{1}{2}, \frac{1}{2}, 1)_1 + D(3, \frac{1}{2}, \frac{1}{2})_0, \quad (41)$$

with bosonic fields being the graviton $g_{\mu\nu}$ and the massless graviphoton $A_1 + \frac{1}{6}\mathbb{A}_1$. Still at the zeroth level, there is also a LH+RH chiral multiplet

$$\begin{aligned} \mathcal{D}(3, 0, 0)_2 &= D(3\frac{1}{2}, \frac{1}{2}, 0)_1 + D(3, 0, 0)_2 + D(4, 0, 0)_0, \\ \mathcal{D}(3, 0, 0)_{-2} &= D(3\frac{1}{2}, 0, \frac{1}{2})_{-1} + D(3, 0, 0)_{-2} + D(4, 0, 0)_0. \end{aligned} \quad (42)$$

The charged $E_0 = 3$ scalar corresponds to the $m^2 = -3$ scalar $b_0^{m^2=-3}$, while the complex $E_0 = 4$ scalar is the axi-dilaton τ .

At the first Kaluza-Klein level, we have a semi-long LH+RH massive gravitino multiplet

$$\begin{aligned} \mathcal{D}(4\frac{1}{2}, 0, \frac{1}{2})_1 &= D(5\frac{1}{2}, \frac{1}{2}, 1)_1 + D(5, \frac{1}{2}, \frac{1}{2})_0 + D(5, 0, 1)_2 + D(6, 0, 1)_0 \\ &\quad + D(4\frac{1}{2}, 0, \frac{1}{2})_1 + D(5\frac{1}{2}, 0, \frac{1}{2})_{-1}, \\ \mathcal{D}(4\frac{1}{2}, \frac{1}{2}, 0)_{-1} &= D(5\frac{1}{2}, 1, \frac{1}{2})_{-1} + D(5, \frac{1}{2}, \frac{1}{2})_0 + D(5, 1, 0)_{-2} + D(6, 1, 0)_0 \\ &\quad + D(4\frac{1}{2}, \frac{1}{2}, 0)_{-1} + D(5\frac{1}{2}, \frac{1}{2}, 0)_1. \end{aligned} \quad (43)$$

The bosonic field content is an $\text{SL}(2, \mathbb{R})$ doublet of $m^2 = 8$ ($E_0 = 5$) vectors b_1^i , a charged $m^2 = 9$ ($E_0 = 5$) anti-symmetric tensor q_2 , and a doublet of $m^2 = 16$ ($E_0 = 6$) anti-symmetric tensors b_2^i .

At the second Kaluza-Klein level, we have a massive vector multiplet

$$\begin{aligned} \mathcal{D}(6, 0, 0)_0 &= D(7, \frac{1}{2}, \frac{1}{2})_0 + D(6\frac{1}{2}, \frac{1}{2}, 0)_{-1} + D(6\frac{1}{2}, 0, \frac{1}{2})_1 + D(7\frac{1}{2}, 0, \frac{1}{2})_{-1} + D(7\frac{1}{2}, \frac{1}{2}, 0)_1 \\ &\quad + D(6, 0, 0)_0 + D(7, 0, 0)_{-2} + D(7, 0, 0)_2 + D(8, 0, 0)_0. \end{aligned} \quad (44)$$

n	Multiplet	State	Field
0	supergraviton	$D(4, 1, 1)_0$ $D(3, \frac{1}{2}, \frac{1}{2})_0$	$g_{\mu\nu}$ $A_1 + \frac{1}{6}\mathbb{A}_1$
0	LH+RH chiral	$D(3, 0, 0)_{\pm 2}$ $D(4, 0, 0)_0 + D(4, 0, 0)_0$	$b_0^{m^2=-3}$ τ
1	LH+RH massive gravitino	$D(5, \frac{1}{2}, \frac{1}{2})_0 + D(5, \frac{1}{2}, \frac{1}{2})_0$ $D(5, 0, 1)_2 + D(5, 1, 0)_{-2}$ $D(6, 0, 1)_0 + D(6, 1, 0)_0$	b_1^i q_2 b_2^i
2	massive vector	$D(7, \frac{1}{2}, \frac{1}{2})_0$ $D(6, 0, 0)_0$ $D(7, 0, 0)_{\pm 2}$ $D(8, 0, 0)_0$	\mathbb{A}_1 σ $b_0^{m^2=21}$ ρ

TABLE II: Identification of the bosonic states in the Kaluza-Klein spectrum with the linearized modes in the reduction.

The massive $E_0 = 7$ vector is the $m^2 = 24$ mode \mathbb{A}_1 . The real $E_0 = 6$ and $E_0 = 8$ scalars are the $m^2 = 12$ squashing and $m^2 = 32$ breathing modes, σ and ρ , respectively. The charged $E_0 = 7$ scalar is $b_0^{m^2=21}$ with $m^2 = 21$. This identification of the linearized fields with the Kaluza-Klein modes is shown in Table II.

For the case of IIB supergravity on S^5 , is interesting to note that these fields lie at the lowest level of the massive trajectories in the Kaluza-Klein mode decomposition of the $D = 10$ fields [31, 32]. We note that the massive Kaluza-Klein tower is built out of scalar, vector and tensor harmonics on S^5 , and the lowest harmonics generally have simple behavior on the internal sphere coordinates. For example, the lowest scalar harmonic is the constant mode on the sphere, while the lowest vector harmonics generate the Killing vectors on the sphere. It is presumably the simplicity of the lowest harmonics that allows the truncation to be consistent, even at the non-linear level.

Although the harmonics on SE_5 are more involved (see *e.g.* [51] for the case of $T^{1,1}$), it is clear that the decomposition (15) and (22) of the $D = 10$ fields in terms of invariant tensors on SE_5 is equivalent to the truncation to the lowest harmonics on the sphere. This appears to be an essential feature guaranteeing the consistency of the massive truncation,

and hence we do not expect to be able to keep any additional multiplets in the Kaluza-Klein tower beyond the $n = 2$ level.

IV. FURTHER TRUNCATIONS

In order to make a connection with previous results on massive consistent truncations of IIB supergravity, we note that the semi-long LH+RH massive gravitino multiplet at the first Kaluza-Klein level may be truncated out by setting

$$b_1^i = 0, \quad b_2^i = 0, \quad q_2 = 0. \quad (45)$$

It is easy to see that this truncation is consistent, since the respective equations of motion for q_2 in (23) and g_2^i and g_3^i in (25) are trivially satisfied in this case. The resulting $D = 5$ Lagrangian takes the form

$$\begin{aligned} \mathcal{L} = & R * 1 + (24e^{2A-2B} - 4e^{5A+3C} - \frac{1}{2}e^{8A}(4 + \phi_0)^2) * 1 - \frac{28}{3}dB \wedge *dB - \frac{8}{3}dB \wedge *dC \\ & - \frac{4}{3}dC \wedge *dC - \frac{1}{2\tau_2^2}d\tau \wedge *d\bar{\tau} - \frac{1}{2}e^{2C-2A}F_2 \wedge *F_2 - e^{A-C}(F_2 + \frac{1}{4}\mathbb{F}'_2) \wedge *(F_2 + \frac{1}{4}\mathbb{F}'_2) \\ & - \frac{1}{2}e^{-8B}[\mathbb{A}'_1 - \frac{2i}{3}\epsilon_{ij}(f_0^i\bar{f}_1^j - \bar{f}_0^i f_1^j)] \wedge *[\mathbb{A}'_1 - \frac{2i}{3}\epsilon_{ij}(f_0^i\bar{f}_1^j - \bar{f}_0^i f_1^j)] \\ & - 2\mathcal{M}_{ij}[e^{5A-C}(f_0^i\bar{f}_0^j + \bar{f}_0^i f_0^j) * 1 + e^{-4B}(f_1^i \wedge *f_1^j + \bar{f}_1^i \wedge *f_1^j)] \\ & - A_1 \wedge (F_2 + \frac{1}{4}\mathbb{F}'_2) \wedge (F_2 + \frac{1}{4}\mathbb{F}'_2), \end{aligned} \quad (46)$$

where

$$f_0^i = 3ib_0^i, \quad f_1^i = Db_0^i, \quad \phi_0 = -\frac{2i}{3}\epsilon_{ij}(f_0^i\bar{f}_0^j - \bar{f}_0^i f_0^j). \quad (47)$$

A further truncation to the massless $\mathcal{N} = 2$ supergravity sector may be obtained by setting

$$b_0^i = 0, \quad B = 0, \quad C = 0, \quad \mathbb{A}_1 = 0, \quad (48)$$

along with taking a constant background for the axi-dilaton, $\tau = \tau_0$. This leaves only $g_{\mu\nu}$ and A_1 , and yields the standard Lagrangian for the bosonic fields of minimal gauged supergravity

$$\mathcal{L} = R * 1 + 12g^2 * 1 - \frac{1}{2}F_2 \wedge *F_2 - \frac{1}{3\sqrt{3}}A_1 \wedge F_2 \wedge F_2, \quad (49)$$

where we have rescaled the graviphoton, $A_1 \rightarrow \frac{1}{\sqrt{3}}A_1$, so that it has a canonical kinetic term, and where we have restored the dimensionful gauged supergravity coupling g .

A. Truncation to the zeroth Kaluza-Klein level

Beyond the truncation to minimal supergravity discussed above, the first nontrivial truncation involves keeping only the lowest Kaluza-Klein level fields $\{\tau, b_0^{m^2=-3}\}$ dynamical. In what follows we will denote $b_0^{m^2=-3}$ simply as b so that $(b_0^1, b_0^2) = (b, \tau b)$. This truncation is not as simple as setting all other fields to zero, as the equations of motion demand certain constraints to be satisfied. For this case we start with the Lagrangian (46), obtained by setting $b_2^i = b_1^i = q_2 = 0$. We then impose the constraints

$$b_0^{m^2=21} = 0, \quad e^{4B} = e^{-4C} = 1 - 4\tau_2|b|^2, \quad \mathbb{A}_1 = -4i\tau_2(bD\bar{b} - \bar{b}Db) + 4|b|^2 d\tau_1. \quad (50)$$

These in turn imply that

$$\phi_0 = -24\tau_2|b|^2, \quad p_2 = -dA_1. \quad (51)$$

To guarantee consistency, we have to check four constraints from the equations of motion (the B , C , f_0^i , and combined Maxwell Equation). They are all verified to hold identically, and hence the truncation to the supergravity plus the LH+RH chiral multiplet is consistent. The Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & R * 1 + [24(1 - 3\tau_2|b|^2)e^{-4B} - 4e^{-8B} - \frac{1}{2}e^{-8B}(4 + \phi_0)^2] * 1 - 8dB \wedge *dB \\ & - \frac{3}{2}F_2 \wedge *F_2 - \frac{1}{2}e^{-8B}\mathbb{A}_1 \wedge *\mathbb{A}_1 - 8e^{-4B}\tau_2 Db \wedge *D\bar{b} - 2ie^{-4B}(\bar{b}Db \wedge *d\bar{\tau} - bD\bar{b} \wedge *d\tau) \\ & - \frac{1}{2\tau_2}(1 + 8e^{-4B}\tau_2|b|^2)d\tau \wedge *d\bar{\tau} - A_1 \wedge F_2 \wedge F_2. \end{aligned} \quad (52)$$

This expression can be simplified by defining $\lambda \equiv 4\tau_2|b|^2$, giving

$$\begin{aligned} \mathcal{L} = & R * 1 + \frac{6(2 - 3\lambda)}{(1 - \lambda)^2} * 1 - \frac{d\lambda \wedge *d\lambda}{2(1 - \lambda)^2} - \frac{(1 + \lambda)d\tau \wedge *d\bar{\tau}}{2(1 - \lambda)\tau_2^2} - \frac{3}{2}F_2 \wedge *F_2 - \frac{\mathbb{A}_1 \wedge *\mathbb{A}_1}{2(1 - \lambda)^2} \\ & - \frac{8\tau_2 Db \wedge *D\bar{b}}{1 - \lambda} - \frac{2i}{1 - \lambda}(\bar{b}Db \wedge *d\bar{\tau} - bD\bar{b} \wedge *d\tau) - A_1 \wedge F_2 \wedge F_2. \end{aligned} \quad (53)$$

If we further truncate the model by setting $\tau = ie^{-\phi_0} = ig_s^{-1}$, which is consistent with the equation of motion for τ given in (8), this reproduces the model used in [16] to describe a holographic superconductor using a $m^2 = -3$ and $q = 2$ charged scalar. If we denote $b = \sqrt{g_s}f e^{i\theta}$, the truncated Lagrangian reads

$$\begin{aligned} \mathcal{L} = & R * 1 - \frac{3}{2}F_2 \wedge *F_2 - A_1 \wedge F_2 \wedge F_2 \\ & + 12 \frac{(1 - 6f^2)}{(1 - 4f^2)^2} * 1 - 8 \frac{df \wedge *df}{(1 - 4f^2)^2} - 8f^2 \frac{(d\theta - 3A_1) \wedge *(d\theta - 3A_1)}{(1 - 4f^2)^2}. \end{aligned} \quad (54)$$

A further redefinition $f = \frac{1}{2} \tanh \frac{\eta}{2}$ then reproduces the Lagrangian given in [16].

B. Truncation to the second Kaluza-Klein level

Starting with the Lagrangian (46) with $b_2^i = b_1^i = q_2 = 0$, it is possible to retain the $b_0^{m^2=21}$ scalar by setting $b_0^{m^2=-3} = 0$. In this case, we first let $b_0^2 = \bar{\tau}b_0^1$ and define $b_0^1 = \sqrt{g_s}he^{i\xi}$, so that (h, ξ) describe the $m^2 = 21$ scalar. Again, the scalar equations of motion lead to constraints, and in particular the first equation in (25) yields the equation of motion for h and ξ as well as

$$d(e^{3A+C} * d\tau) + ie^{3A+C} \frac{1}{\tau_2} d\tau \wedge *d\tau = 0. \quad (55)$$

This is simply the τ equation of motion without any sources, and the simplest thing to do is to set τ to be constant, $\tau = ie^{-\phi_0} = ig_s^{-1}$. The remaining field content is then $\{g_{\mu\nu}, A_1, \rho, \sigma, b_0^{m^2=21}, \mathbb{A}_1\}$, corresponding to the supergravity multiplet coupled to the massive vector multiplet. It is now straightforward to complete the truncation, and the Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & R * 1 + \left(24e^{-\frac{16}{3}\rho-\sigma} - 4e^{-\frac{16}{3}\rho-6\sigma} - 8e^{-\frac{40}{3}\rho}(1+6h^2)^2 \right) * 1 - \frac{40}{3}d\rho \wedge *d\rho - 5d\sigma \wedge *d\sigma \\ & - \frac{1}{2}e^{\frac{16}{3}\rho-4\sigma} F_2 \wedge *F_2 - e^{-\frac{8}{3}\rho+2\sigma} (F_2 + \frac{1}{4}\mathbb{F}'_2) \wedge * (F_2 + \frac{1}{4}\mathbb{F}'_2) \\ & - \frac{1}{2}e^{-8\rho-4\sigma} (\mathbb{A}'_1 + 8h^2\Gamma) \wedge * (\mathbb{A}'_1 + 8h^2\Gamma) - A_1 \wedge (F_2 + \frac{1}{4}\mathbb{F}'_2) \wedge (F_2 + \frac{1}{4}\mathbb{F}'_2) \\ & - 8(e^{-4\rho-2\sigma} dh \wedge *dh + e^{-4\rho-2\sigma} h^2\Gamma \wedge *\Gamma + e^{-\frac{28}{3}\rho+2\sigma} h^2 * 1), \end{aligned} \quad (56)$$

where we have defined $\Gamma = d\xi - 3A_1$.

We can further truncate this by removing the $m^2 = 21$ scalar (*i.e.* by setting $h = \xi = 0$), giving the Lagrangian

$$\begin{aligned} \mathcal{L} = & R * 1 + \left(24e^{-\frac{16}{3}\rho-\sigma} - 4e^{-\frac{16}{3}\rho-6\sigma} - 8e^{-\frac{40}{3}\rho} \right) * 1 - \frac{40}{3}d\rho \wedge *d\rho - 5d\sigma \wedge *d\sigma \\ & - \frac{1}{2}e^{\frac{16}{3}\rho-4\sigma} F_2 \wedge *F_2 - e^{-\frac{8}{3}\rho+2\sigma} (F_2 + \frac{1}{4}\mathbb{F}'_2) \wedge * (F_2 + \frac{1}{4}\mathbb{F}'_2) - \frac{1}{2}e^{-8\rho-4\sigma} \mathbb{A}'_1 \wedge *\mathbb{A}'_1 \\ & - A_1 \wedge (F_2 + \frac{1}{4}\mathbb{F}'_2) \wedge (F_2 + \frac{1}{4}\mathbb{F}'_2), \end{aligned} \quad (57)$$

which corresponds to the $m^2 = 24$ massive vector field truncation of [18].

C. Non-supersymmetric truncations

All the truncations we have listed so far have field content which fills the bosonic sector of AdS_5 supermultiplets and so are presumably supersymmetric truncations. It is also useful to

consider truncations which contain dynamical fields belonging to different supermultiplets, without keeping the entire multiplet. In this sense these truncations are not supersymmetric, although they are perfectly consistent truncations and solutions of the ten-dimensional equations of motion. For these truncations, we start with the complete Lagrangian given in (30) and (31).

1. Massive vector field

The first non-supersymmetric truncation we will discuss involves keeping the $m^2 = 8$ vector field, b_1^i , and has already been noted in [18]. The field content in this truncation consists of one component of b_1^i (denoted b_1), τ_2 , ρ , σ and $g_{\mu\nu}$. Note that the graviphoton is turned off here so that even at the lowest level this cannot be supersymmetric. Furthermore, by keeping only one component of b_1^i , the τ equation of motion demands that we must set $\tau_1 = 0$. With this field content, the $D = 10$ constraints (24) are trivially satisfied with $\phi_0 = 0$ and $p_2 = 0$, and the Lagrangian (30) becomes [18]

$$\begin{aligned} \mathcal{L} = & R * 1 + (24e^{-\frac{16}{3}\rho-\sigma} - 4e^{-\frac{16}{3}\rho-6\sigma} - 8e^{-\frac{40}{3}\rho}) * 1 - \frac{40}{3}d\rho \wedge *d\rho - 5d\sigma \wedge *d\sigma \\ & - \frac{1}{2\tau_2^2}d\tau_2 \wedge *d\tau_2 - \frac{1}{2}\tau_2 e^{\frac{4}{3}\rho+4\sigma}db_1 \wedge *db_1 - 4\tau_2 e^{-4\rho-2\sigma}b_1 \wedge *b_1. \end{aligned} \quad (58)$$

2. Complex massive anti-symmetric tensor

We can also truncate to theories containing the $m^2 = 9$ complex anti-symmetric tensor field q_2 . The field content here is given by, q_2 , \mathbb{A}_1 , B , C , τ , $g_{\mu\nu}$ and A_1 . The $D = 10$ constraints become $\phi_0 = 0$ and $p_2 = -dA_1 - \frac{1}{4}d\mathbb{A}_1$. All the other equations of motion are either satisfied by setting the rest of the fields to zero or can be derived from the Lagrangian

$$\begin{aligned} \mathcal{L} = & R * 1 + (24e^{-\frac{16}{3}\rho-\sigma} - 4e^{-\frac{16}{3}\rho-6\sigma} - 8e^{-\frac{40}{3}\rho}) * 1 - \frac{40}{3}d\rho \wedge *d\rho - 5d\sigma \wedge *d\sigma \\ & - \frac{1}{2}e^{\frac{16}{3}\rho-4\sigma}F_2 \wedge *F_2 - e^{-\frac{8}{3}\rho+2\sigma}(p_2 \wedge *p_2 + 4q_2 \wedge *\bar{q}_2) - \frac{1}{2\tau_2^2}d\tau \wedge *d\bar{\tau} \\ & - \frac{1}{2}e^{-8\rho-4\sigma}\mathbb{A}_1 \wedge *\mathbb{A}_1 + \frac{2i}{3}(q_2 \wedge d\bar{q}_2 - \bar{q}_2 \wedge dq_2) - A_1 \wedge p_2 \wedge p_2 - 4A_1 \wedge q_2 \wedge \bar{q}_2. \end{aligned} \quad (59)$$

Note that it is consistent to further truncate to a constant axi-dilaton $\tau = \tau_0$.

3. Real massive anti-symmetric tensor

Along similar lines to the case above for a massive vector field, we can set $A_1 = 0$ and make a truncation including the $m^2 = 16$ real anti-symmetric tensor doublet b_2^i by keeping only the graviton coupled to b_2^i , τ , ρ and σ . Again, the equations of motion for the other fields are trivially satisfied and the constraints are also trivial $\phi_0 = 0$ and $p_2 = 0$. This leaves the Lagrangian

$$\begin{aligned} \mathcal{L} = & R * 1 + (24e^{-\frac{16}{3}\rho-\sigma} - 4e^{-\frac{16}{3}\rho-6\sigma} - 8e^{-\frac{40}{3}\rho}) * 1 - \frac{40}{3}d\rho \wedge *d\rho - 5d\sigma \wedge *d\sigma \\ & - \frac{1}{2\tau_2^2}d\tau \wedge *d\bar{\tau} - \frac{1}{2}e^{\frac{20}{3}\rho}\mathcal{M}_{ij}db_2^i \wedge *db_2^j + 2\epsilon_{ij}b_2^i \wedge *db_2^j. \end{aligned} \quad (60)$$

As in the previous truncation it is consistent to further truncate to $\tau = \tau_0$.

V. DISCUSSION

In the above, we have examined massive reductions of 10-dimensional IIB supergravity on Sasaki-Einstein manifolds. By utilizing the structure of SE_5 , we have given a general decomposition of the IIB fields based on the invariant tensors associated with the internal manifold. The field content obtained in five-dimensions completes the bosonic sector of various AdS_5 supermultiplets, and in particular they fill out the lowest Kaluza-Klein tower up to the breathing mode supermultiplet. This proves, at least at the level of the bosonic fields, the conjecture raised in [46] that a consistent massive truncation may be obtained by truncating to the singlet sector on the Kahler-Einstein base B (which is CP^2 for the squashed S^5) and further restricting to the level of the breathing mode multiplet and below.

As suggested at the end of Section III, it is this truncation to constant modes on the base B that ensures the consistency of the reduction. In a sense, this is a generalization of the old consistency criterion of restricting to singlets of the internal isometry group, except that here restricting to singlets of an appropriate subgroup turned out to be sufficient. For this reason, we believe it is not that the breathing mode is special in itself which allows for a consistent truncation retaining its supermultiplet, but rather that in the examples given here and in [13], the breathing mode superpartners so happen to be the lowest harmonics in their respective Kaluza-Klein towers. It is an unusual feature of Kaluza-Klein compactifications on curved internal spaces that states originating from different levels of the harmonic expansion

may combine into a single supermultiplet. Thus, while the breathing mode is always the lowest state in its tower (being a constant mode on the internal space), its superpartners may carry excitations on the internal space. This does not occur for the $\mathcal{N} = 2$ compactification of IIB supergravity on SE_5 (nor does it for $D = 11$ supergravity on SE_7). However, in extended theories, such as IIB supergravity on the round S^5 , the superpartners will involve non-trivial harmonics. In particular, the $\mathcal{N} = 8$ superpartners to the breathing mode include a massive spin-2 excitation of the graviton involving the second harmonic (d-waves) on the sphere. Thus we believe it to be unlikely that an $\mathcal{N} = 8$ massive truncation with the breathing mode multiplet will be consistent.

Consistent truncations of the type discussed here have recently been of particular interest in the growing literature on AdS/CFT applications to condensed matter systems. Until recently a strictly phenomenological approach has been taken in this area. In these systems the inclusion of a scalar condensate is required in the gravity theory to source an operator whose expectation value acts as an order parameter describing superconductor/superfluid phase transitions in the strongly coupled system. In the phenomenological approach, the origin of this scalar and its properties have not been of immediate interest; rather the general behavior was determined and many interesting similarities to real condensed matter systems have been noted. However, this approach lacks strong theoretical control in that systems are described by a set of free parameters which can be tuned to provide the property of interest. Recently there has been some work to embed these models in UV complete theories, where the parameters are no longer free but are determined by the underlying features of the theory, such as an origin in string theory. The discussion here has put these reductions into a more general framework and gives further examples of UV complete systems whose duals may have useful applications in the AdS/CMT correspondence.

Given that the fields in these truncations fall into specific supermultiplets it is an obvious and relevant question to discuss their fermionic partners. This would involve reducing the supersymmetry variations and fermion equations in ten-dimensions down to five-dimensions and determining the complete supersymmetric action of these truncations. This is also of interest in terms of AdS/CMT where there has been much interest in describing fermion behavior in condensed matter systems such as the Fermi-liquid theory using the holographic correspondence. In particular, the full supersymmetric action could give us examples of specific interactions studied in these systems coupling scalar condensates to the fermionic

excitations [52–54]. We leave the study of the fermionic modes and connections to condensed matter systems to future work.

Acknowledgments

We wish to thank I. Bah, M. Duff, J. Gauntlett and D. Vaman for stimulating discussions. This work was supported in part by the US Department of Energy under grant DE-FG02-95ER40899.

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